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Normalized sensitivity measures for leaf area index estimation using three-band spectral vegetation indices

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A great number of spectral vegetation indices (SVIs) have been developed to estimate key biophysical parameters such as leaf area index (LAI). Considerable interest is often given to the local optimization, performance analysis and sensitivity of each spectral band and SVI for LAI estimation given that several confounding factors are present. In this regard, inclusion of shortwave infrared (SWIR) reflectance in traditionally near-infrared (NIR)-red (R)-based SVIs has played a great role for local optimization and increased sensitivity of SVIs to LAI. This study presents the enhanced and normalized sensitivity functions for evaluating (1) the sensitivity of each spectral band and SVI to LAI and (2) the generic performance analysis of empirical model to estimate LAI based on the SVIs. Several alternatives for three-band (NIR-R-SWIR) SVI modifications have been recommended and proven to be simplistic and unbiased way of local optimization.

1. Introduction

1.1 *Three-band spectral vegetation index*

A spectral vegetation index (SVI) is a numerical indicator used to measure the amount and vigour of photosynthetic materials on land surface, usually formed from arithmetic combinations of two or more spectral bands. SVI remains the most valuable quantitative vegetation monitoring tool when the photosynthetic capacity of the land surface needs to be studied for various phenomena. The simplest form of SVI is a ratio between near-infrared (NIR) and red (R) reflectances. A number of derivatives and alternatives to commonly used SVI such as normalized difference vegetation index (NDVI) proposed in an attempt to include intrinsic corrections for one or more perturbing problems encountered by using only simple NIR–R indices. It was not until the mid-1990s, however, that the use of shortwave infrared (SWIR) reflectance in formulating SVI was proposed to estimate the vegetation water content and biophysical parameters (e.g. Nemani *et al.* 1993, Gao 1996). Including the SWIR reflectance in the simple NIR–R ratio, for example, improved the SVI–leaf area index (LAI) relationship (Brown *et al.* 2000), owing to the reduction in the variation of reflectance caused by the heterogeneous background and canopy cover. Above and beyond, the SWIR has the advantage of being sensitive to vegetation while being less susceptible to aerosol because of its larger wavelength compared to the radius of most aerosols.

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This will circumvent the uncertainty associated with atmospheric correction of remotely sensed reflectance. Thus, using the three spectral bands (NIR, R and SWIR) in the formulation of SVI will enhance the estimation performance particularly for retrieving key biophysical parameters by taking into account the confounding factors such as the background adjustment and atmospheric effects on obtained reflectance.

Although adding SWIR reflectance to classical SVIs was proven to improve the SVI–LAI relationships, it has fundamental problem associated with soil moisture variations such as could be found on irrigated cropland in contrast to its ascribed merit (e.g. Brown *et al.* 2000). In addition to this, SWIR adjustments on, for example, NDVI and simple NIR–R ratio are commonly done based on rescaled difference of local SWIR maximum and minimum reflectances. This approach is arbitrary and may not hold true for larger landscape because the soil moisture content may vary independently. Therefore, the objectives of this letter are (1) to develop normalized sensitivity and performance evaluation functions for SVI–LAI relationship, (2) to evaluate the enhanced merit of adding SWIR on NIR–R based SVIs and (3) to simplify the SWIR adjustments without arbitrary decisions on the dataset simulated with extreme soil moisture contents.

1.2 *Normalized sensitivity functions*

The statistical fitting of biophysical parameters such as LAI with SVI is mainly carried out using least-squares regression, whereby their generic form is expressed as LAI = *f*(SVI). Theoretically, the function should have been formulated in reverse order such that SVI is the function of LAI, that is, SVI = *f*(LAI). Therefore, hereafter the independent variable LAI is expressed as X and the dependent variable (reflectance or SVI) as Y, unless stated otherwise. Traditionally, regression techniques are used to evaluate the sensitivity of SVI to biophysical parameters with global sensitivity statistics, such as the coefficient of determination (R^2), and root mean squared error (RMSE) as goodness-of-fit measures. However, R^2 and other global goodness-of-fit measures have drawbacks because they are functions of the range and variance of samples. To this regard, several studies proposed local sensitivity analysis of SVI–LAI (see brief review in Ji and Peters (2007)). As argued by these authors, all of the sensitivity measures either do not account for random variation of $\mathrm{d}\hat{Y}$ on $\mathrm{d}X$ as $\hat{Y}$ is the estimate derived from the fitted model or need large amount of field measurements.

Mathematical sensitivities (S, also in the form of model output derivatives) are a straightforward implementation of local sensitivity concept. The absolute sensitivity of Y to variations in the parameter X is

$$S_X^Y = \frac{\mathrm{d}Y}{\mathrm{d}X}, \tag{1}$$

where S_X^Y gives the rate of change of Y as a function of change in X. Here, (1) S_X^Y is theoretically only valid for infinitesimally small variations; (2) S_X^Y is essentially only valid for linear relationships or models; (3) if a model is used such that $\mathrm{d}\hat{Y}/\mathrm{d}X$ instead of $\mathrm{d}Y/\mathrm{d}X$, it does not account for random variations of $\hat{Y}$ on X; (4) S_X^Y is sensitive for the unit and magnitude variations of various Xs as in the cases of different SVIs and reflectances in various spectral bands; (5) S_X^Y determines changing only a single X at a time; and (6) S_X^Y has a problem of division by 0.

If a model independent of unit is used as cases of SVIs, the normalized sensitivity function (N_X^Y) can be calculated in the following way:

$$N_X^Y = \frac{\bar{X}}{\bar{Y}} \lim_{\Delta X \to 0} \frac{\Delta Y}{\Delta X} = \frac{\bar{X}}{\bar{Y}} \frac{\mathrm{d}Y}{\mathrm{d}X} = \frac{\mathrm{d}\ \ln Y}{\mathrm{d}\ \ln X}, \tag{2}$$

where $\bar{X}$ is the nominal (or central, if a range is known) value of factor X and $\bar{Y}$ is the value taken by Y when all X factors are at their nominal value. The normalized sensitivity function N_X^Y can be interpreted as the approximate percentage change in Y for a 1% increase in the parameter X. Equation (2) can be interpreted as a cause-and-effect relationship between the LAI variations and the resulting fractional change in SVI (e.g. $\mathrm{d}\,(\mathrm{SVI})/(\mathrm{SVI}) = N_{\mathrm{LAI}}^{\mathrm{SVI}}\,\mathrm{d}\,(\mathrm{LAI})/(\mathrm{LAI})$). The N_X^Y is particularly very important to study the major explaining variable regardless of the magnitude of the differentials in the independent variable. For example, in the vegetation growing on very bright soil where NIR reflectance usually reaches the saturation point at low LAI values, the reflectance in R spectral band is the major factor contributing to SVI for changing LAI although the differential of the reflectance in R is relatively small. Therefore, N_X^Y is a robust indicator for explaining the proportional change rather than the magnitude of the differential as does the simple mathematical sensitivity index shown in equation (1).

To consider variation in several parameters such as spectral bands changing simultaneously with certain biophysical parameters is of interest. Change in Y due to small changes in all X_j parameters ($j = 1, 2, 3, \ldots, n$) can be expressed with the total differential:

$$\mathrm{d}Y = \sum_{j=1}^{n} \frac{\partial Y}{\partial X_j} \mathrm{d}X_j. \tag{3}$$

The above equation is the equivalent of the multivariable first-order Taylor series expansion when higher order terms are ignored (Abramowitz and Stegun 1970). To normalize the above equation for unit or magnitude variations, we can modify equation (3) as follows:

$$\frac{\mathrm{d}Y}{\bar{Y}} = \sum_{j=1}^{n} \left(\frac{\partial Y}{\partial X_j} \frac{\bar{X}_j}{\bar{Y}} \right) \frac{\mathrm{d}X_j}{\bar{X}_j}. \tag{4}$$

Usually, increments are more instructive, therefore equation (4) can be approximated as

$$\frac{\Delta Y}{\bar{Y}} = \sum_{j=1}^{n} N_{X_j}^{Y} \frac{\Delta X_j}{\bar{X}_j}. \tag{5}$$

Therefore the multi-parameter normalized sensitivity function ($\overline{M}$) will be:

$$\overline{M_n} = \sum_{j=1}^{n} \left| N_{X_j}^{Y} \right| \frac{\Delta X_j}{\bar{X}_j}, \tag{6}$$

where the summation is over independent variables, n. It is an absolute summation, because there is no interaction effect to cancel out each other's contribution, nor correlation among the independent variables. Therefore, equation (6) is only valid for independent variables which do not exhibit correlation.

A crude way to analyse the sensitivity of the estimate $\hat{Y}$ (in the form of model output instead of Y as equation (2)) to X is $S_X^{\hat{Y}} = \mathrm{d}\hat{Y}/\mathrm{d}X$. This equation is the same as equation (1) except that the sensitivity is carried out for the estimate ($\hat{Y}$) instead of the original dependent variable (Y). This can be modified for the unit and magnitude variations as follows:

$$N_X^{\hat{Y}} = \frac{\bar{X}}{\bar{\hat{Y}}} \lim_{\Delta X \to 0} \frac{\Delta \hat{Y}}{\Delta X} = \frac{\bar{X}}{\bar{\hat{Y}}} \frac{\mathrm{d}\hat{Y}}{\mathrm{d}X} = \frac{\mathrm{d} \ln \hat{Y}}{\mathrm{d} \ln X} = \frac{\bar{X}}{\bar{\hat{Y}}} \hat{Y}', \tag{7}$$

where $\hat{Y}'$ is the first derivative that reflects the sensitivity of X to $\hat{Y}$, not to Y. The problem with this equation is that it does not incorporate the random errors of the $\hat{Y}$ estimates. Ji and Peters (2007) modified the non-normalized version of equation (7) by adding the random error of the estimate as

$$S_X^{\hat{Y},\sigma} = \frac{\hat{Y}'}{\sigma_{\hat{Y}}} = \frac{\mathrm{d}\hat{Y}}{\mathrm{d}X} \Big/ \sigma_{\hat{Y}}. \tag{8}$$

$\sigma_{\hat{Y}}$ is standard error of $\hat{Y}$ or asymptotic standard error $\hat{Y}$. Although, the $S_X^{\hat{Y},\sigma}$ value is irrelevant to the magnitude of the Y (SVI), it is dependent on the unit or magnitude of the X (LAI). However, there are many occasions where we may have to calculate the sensitivity of LAI to SVIs, the inverse order shown above. In such cases, the $S_X^{\hat{Y},\sigma}$ is not the appropriate performance index. We can normalize equation (8) for unit, magnitude and dynamic range differences as follows:

$$N_X^{\hat{Y},\sigma} = \frac{\mathrm{d}\hat{Y}}{\mathrm{d}X} \frac{\bar{X}}{\bar{\hat{Y}}} \Big/ \sigma_{\hat{Y}} = \frac{\mathrm{d} \ln \hat{Y}}{\mathrm{d} \ln X} \Big/ \sigma_{\hat{Y}} \tag{9}$$

Both $S_X^{\hat{Y},\sigma}$ and $N_X^{\hat{Y},\sigma}$ indicate the t-score, the latter value indicating the Student's t-test for the normalized sensitivity function for a given p-value because $t \cong \hat{Y}_i'/\sigma_{\hat{Y}_i}$ (Ji and Peters 2007). However, equations (8) and (9) still assume linear relationship between Y and X; therefore, they may not be robust enough to draw conclusion for non-linear relationships, those expected particularly between SVI and LAI. The other problem with equations (8) and (9) is that whatever perturbation is there in the input datasets, the $S_X^{\hat{Y},\sigma}$, $N_X^{\hat{Y},\sigma}$ and $\sigma_{\hat{Y}}$ remain the same for the same X having varying Y (as can be expected between SVIs and LAIs on heterogeneous soil background conditions). To assess the sensitivity of Y with X, also including the model performance, simple performance analysis such as the sensitivity of the estimate $\hat{Y}$ to Y can be evaluated as $\hat{Y}$ also includes the model performance, that is, the relationship between X and Y:

$$S_{\hat{Y}}^{Y} = \frac{\mathrm{d}Y}{\mathrm{d}\hat{Y}}. \tag{10}$$

Equation 10 indicates the performance of the model, which is used to fit Y with X, and works well for both linear and non-linear relationships. If $S_{\hat{Y}}^{Y}$ is 1, Y is highly sensitive to X, that is, the model fitted between Y and X which are used to estimate $\hat{Y}$ is robust fit. If $S_{\hat{Y}}^{Y}$ is 0, Y is not sensitive to changes of X or the model fit used to estimate $\hat{Y}$ did not show the variability of Y for changes in X. Equation (10) works well for the exceptional conditions where the same X having different Y results in the same $\hat{Y}$, and vice versa results in different $\hat{Y}$. Any values that are close to 0 or very large positive and negative numbers indicate the poor performance of the model used to fit Y with X or poor relationship between Y and X. We can associate the standard error of the estimates for equation (10), the same way as equation (8). If we fit Y with estimated $\hat{Y} = (Y = f(\hat{Y}))$, the $\sigma_{\hat{Y}}$ from the model fit of Y versus $\hat{Y}$ is exactly the same with $\sigma_{\hat{Y}}$ of the original model used to fit Y with X. The standard error of the estimate for ith row is calculated as follows:

$$\begin{aligned} \sigma_{\hat{Y}i} &= \sqrt{\sigma^2 \mathbf{X'}_i (\mathbf{X'X})^{-1} \mathbf{X}_i} \text{ for the linear model and} \\ &= \sqrt{\sigma^2 \hat{\mathbf{Y}}'_i (\hat{\mathbf{Y}}'\hat{\mathbf{Y}})^{-1} \hat{\mathbf{Y}}_i} \text{ for the non-linear model,} \end{aligned} \tag{11}$$

where σ^2 is the mean squared error; $\mathbf{X}$ and $\hat{\mathbf{Y}}$ in this case are matrices of the independent variables for the dependent variable X; $\mathbf{X}_i$ and $\hat{\mathbf{Y}}_i$ are the ith rows of $\mathbf{X}$ and $\hat{\mathbf{Y}}$, respectively. Note that in equation (11) $\hat{\mathbf{Y}}'$ represents matrix transpose, rather than the first derivative $\hat{Y}'$. This is also the simplest way of calculating the standard error of the estimate ($\sigma_{\hat{Y}}$) particularly for non-linear model, which is mathematically complex and is not available with common statistical packages. Associating $\sigma_{\hat{Y}}$ with $S^Y_{\hat{Y}}$ is beyond the scope of this study.

2. Empirical test of the sensitivity of three-band SVIs using simulated spectra

2.1 *Data*

The combined PROSPECT leaf optical properties model and SAIL turbid medium canopy bidirectional reflectance model, also referred to as PROSAIL (Jacquemoud *et al.* 2009), were used to evaluate the spectral sensitivity of various SVIs to LAI. These models are the common tools to devise new SVI and evaluate its efficiency for estimating the biophysical or biochemical property of interest. The canopy reflectance was simulated for three wavelength bands of 1-nm bandwidth centred around 645, 834 and 1645 nm, corresponding to R, NIR and SWIR bands, respectively, for a range of plausible input parameters. It is possible that some other combinations of specific position and width of the bands would have provided slightly different SVI values; however, a sensitivity analysis accounting for these specific combinations was beyond the scope of this study. The central bands were selected based on the typical vegetation instruments such as SPOT-HRV, SPOT-HRVIR and SPOT-HRG, and the empirical LAI estimation was assumed to be less sensitive to bandwidth variations. PROSPECT uses the following input parameters: chlorophylls-*a* and *b* content (C_{ab}, μg cm^{-2}), dry biomass content (C_m, g cm^{-2}), equivalent leaf water content (C_w, g cm^{-2}) and mesophyll structure parameter (N); and SAIL uses LAI, leaf angle distribution (LAD), soil reflectance (ρ_s) and external parameters like view zenith (θ_v), Sun zenith (θ_s), view azimuth (φ_v) and Sun azimuth (φ_s) angles. For this analysis, the ranges of the LAI inputs were between 0.25 and 9 by an interval of 0.25. The largest LAI value of 12 representing the highest photosynthetic biomass density was also simulated which was added in the spectral data pool to drive the minimum and maximum values of R and SWIR reflectances (Gonsamo 2010). Three extreme soil backgrounds ranging from dark (wet) to bright (dry) were specified from Bowker *et al.* (1985) and were used to evaluate the SVI–LAI relationships for varying soil spectra, as the major aim of the three-band SVIs was to reduce the sensitivity of LAI to background reflectance. The other input parameters were kept constant: $N = 1.55$, $C_{ab} = 34.24$, $C_w = 0.0137$, $C_m = 0.0045$, spherical LAD, $\theta_v = 0°$, $\theta_s = 30°$, $\varphi_v = 0°$ and $\varphi_s = 0°$. Note from figure 1(*a*) that the dark soil background absorbs more than the fully covered vegetation in SWIR region.

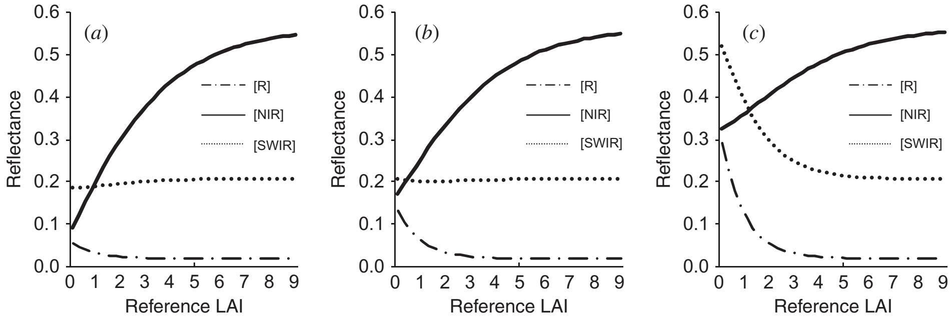

Figure 1. Relationships between LAI and reflectances in three spectral bands simulated for (*a*) dark, (*b*) intermediate and (*c*) bright soil background types.

2.2 *Spectral vegetation indices*

Eight SVIs formulated from R, NIR and SWIR reflectances from simulated datasets used in this study and their definitions are presented in table 1. Maximum and minimum of R and SWIR bands were obtained from the total collection of simulated spectra including the largest LAI (=12) and soil spectra. The basic indices are included for comparative analysis although this study mainly focuses on the three-band SVIs.

SWIR-adjusted simple ratio (SASR) and red-adjusted infrared simple ratio (RAISR) are introduced in this study as a modification of the arbitrarily defined

Table 1. Summary of eight ratio-based spectral vegetation indices used in this analysis.

Acronym: index	Algorithm	Description
NDVI: normalized difference vegetation index	$\frac{\rho_{NIR}-\rho_R}{\rho_{NIR}+\rho_R}$	Basic index
SR: simple ratio	$\frac{\rho_{NIR}}{\rho_R}$	Basic index
ISR: infrared simple ratio	$\frac{\rho_{NIR}}{\rho_{SWIR}}$	Modification of RSR to minimize uncertainty in atmospheric correction (Fernandes *et al.* 2003)
RSR: reduced simple ratio	$\frac{\rho_{NIR}}{\rho_R}\frac{(\rho_{SWIR.max}-\rho_{SWIR})}{(\rho_{SWIR.max}-\rho_{SWIR\,min})}$	Modification of SR to minimize the reflectance variations caused by different background or cover types (Brown *et al.* 2000)
RISR: reduced infrared simple ratio	$\frac{\rho_{NIR}}{\rho_{SWIR}}\frac{(\rho_{R.max}-\rho_R)}{(\rho_{R.max}-\rho_{R\,min})}$	Modification of ISR to minimize the reflectance variations caused by different soil moisture content (Gonsamo and Pellikka 2010)
SADI: SWIR-adjusted difference index	$\frac{\rho_{NIR}-\rho_R}{\rho_{SWIR}}$	Simple SWIR modification of difference index (this study)
SASR: SWIR-adjusted simple ratio	$\frac{\rho_{NIR}-\rho_{NIR}\rho_{SWIR}}{\rho_R}$	Modification of RSR by assigning SWIR max and min to 1 and 0, respectively (this study)
RAISR: red-adjusted infrared simple ratio	$\frac{\rho_{NIR}-\rho_{NIR}\rho_R}{\rho_{SWIR}}$	Modification of RISR by assigning R max and min to 1 and 0, respectively (this study)

ρ_R = red reflectance, ρ_{NIR} = near-infrared reflectance, ρ_{SWIR} = shortwave infrared reflectance. Max and min are maximum and minimum, respectively.

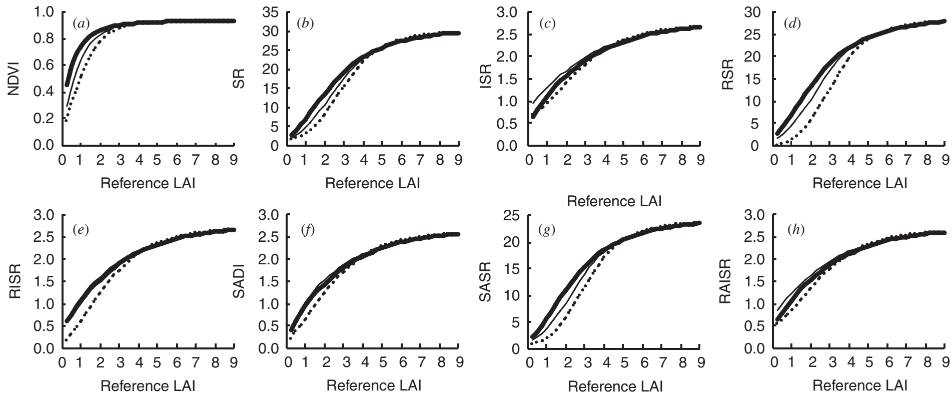

Figure 2. Simulated results of the different soil backgrounds and spectral vegetation indices plotted along the LAI. The formulations of the indices are given in table 1. Dark (bold line), intermediate (solid line) and bright (broken line) soil backgrounds. (*a*) NDVI, (*b*) SR, (*c*) ISR, (*d*) RSR, (*e*) RISR, (*f*) SADI, (*g*) SASR and (*h*) RAISR.

local maximum and minimum values of SWIR and R reflectances for rescaling simple ratio (SR) and infrared simple ratio (ISR), respectively (Brown *et al.* 2000, Fernandes *et al.* 2003, Gonsamo and Pellikka 2010). As commonly implemented, the constant local maximum and minimum are the same over certain landscapes where empirical data are being fitted. To avoid bias and dependency of local land cover histograms (Gonsamo 2010), it is indispensable to devise both SWIR and R adjustment values without subjective decisions. To test this hypothesis, the extreme soil background reflectances which have contrasting trends in SWIR reflectance along the increasing LAI were used in this study (figure 1). The modification resulted in the same trend of relationships with LAI with slight insensitivity for soil spectral variations (see in figure 2 reduced simple ratio (RSR) vs. SASR, reduced infrared simple ratio (RISR) vs. RAISR).

2.3 *Results*

Figure 3 shows the normalized sensitivity of each spectral band and their various combinations on LAI for the three soil background types. The soil background has strong effect on the sensitivity of each of the spectral bands for explaining the variation on LAI. Overall, the reflectance in red spectral region is found to be more sensitive for LAI changes across various soil background types for lower LAI values. The high sensitivity of R in low LAI is largely due to the changing fraction of illuminated background, which is well explained in NDVI. For LAI up to 2–3 where the fraction of vegetation cover reaches maximum for randomly distributed leaves, NDVI and R reflectance show high sensitivity (figures 2(*a*) and 3). The darker the soil the larger the LAI, and the reflectance in NIR alone explains the variability of LAI more than the R and SWIR combined. The normalized sensitivity function shown in equation (2) reveals the relative sensitivity of the spectral bands and their various combinations regardless of the magnitude of their differential changes along the increasing LAI. For example, one can note from figure 1 that the magnitude change of the reflectances along the LAI is high in NIR compared to the R and SWIR regions. However, the sensitivity of the spectral bands is evaluated based on the

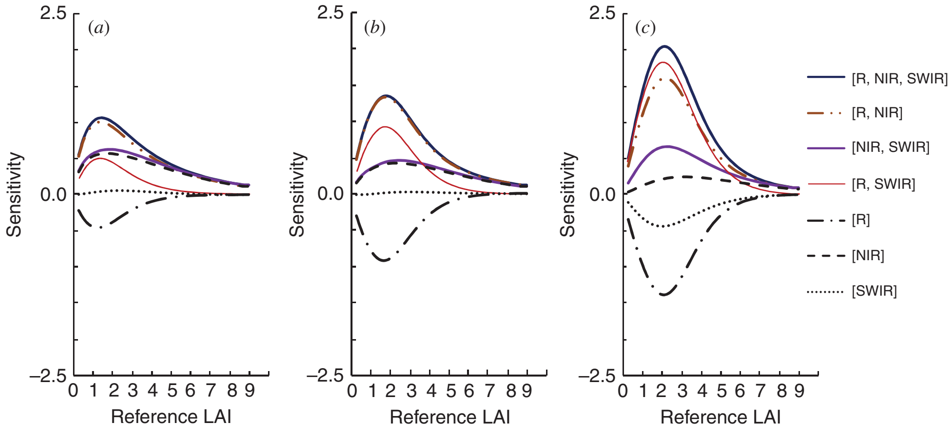

Figure 3. The sensitivity of the three spectral bands using equation (2), and the various combinations of the three spectral bands following the normalized multi-parameter sensitivity function (equation (6)) plotted along LAI for (*a*) dark, (*b*) intermediate and (*c*) bright soil background types.

proportional change along LAI rather than the magnitude, which is well depicted using the normalized sensitivity function (equation (2), figure 3). On very bright soil, R and SWIR perform well, whereas in dark and intermediate soil the contribution of SWIR on commonly used bands (NIR and R) is negligible. The transmittance and reflectance of NIR region are very high from green leaves, therefore the contribution of background to NIR canopy reflectance is relatively large. Consequently, the NIR scattering from background soil contributes significantly to volume reflectance of the vegetation canopy resulting in reduced sensitivity of NIR with decreasing moisture content (figure 3). For darker and intermediate soil background, the latter explaining the most commonly expected soil background in real canopies, the performance of the combination of R and NIR alone is comparable with the performance of all the three spectral bands. However, on very bright soil, the combination of R and SWIR spectral bands outperforms those of the commonly used R and NIR bands because of saturation in NIR band for relatively lower LAI value (figure 3(*c*)).

Depending on the formulation of SVI, one should note that, rather than the magnitude change in reflectance with increasing LAI, the most important attribute is proportional change of reflectance along the various soil backgrounds for increasing LAI. In such a case, SVIs can be formulated in a way to result in the small reflectance change on larger proportional change in SVI. On darker soil with low SWIR reflectance, there is smaller relative change in SWIR reflectance along the increasing LAI because the leaves have comparable reflectance with darker soil (figure 3(*a*) and (*b*)). On the contrary, in very bright soil where there is very high NIR reflectance, there is relatively smaller change of NIR reflectance for increasing LAI because the contribution of the soil to canopy reflectance is at its maximum. At very large LAI values (>6), the change in NIR reflectance is the main driving factor for SVIs to explain the increasing LAI (figure 3).

The relative sensitivity of the eight SVIs used in this study are shown in figure 4 based on the normalized sensitivity function (equation (2)). The most important attribute for SVI in figure 4 is to have consistent trend with LAI on various soil

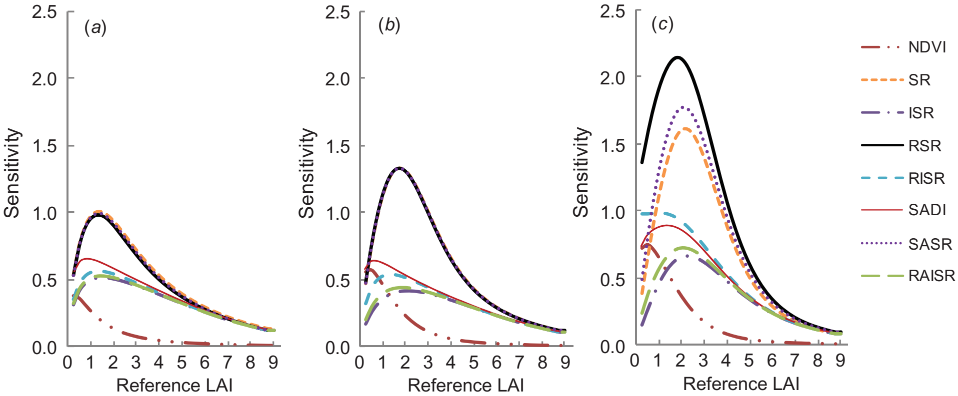

Figure 4. The sensitivity of eight spectral vegetation indices following the normalized sensitivity function (equation (2)) plotted along LAI for (*a*) dark, (*b*) intermediate and (*c*) bright soil background types.

backgrounds because there can be several soil backgrounds in real canopies. Except that of NDVI on all three soil backgrounds and RISR on bright soil, all the SVIs show sigmoidal relationships with increasing LAI where the sensitivity increases until LAI is between 2 and 4, then decreases for the remaining LAI. Although formulated using the same NIR and R spectral bands as SR, NDVI saturates at low level of LAI because small changes on both spectral bands are normalized out when the difference is divided by the sum. On the contrary, as NIR is divided by R for SR, a small change of R has a high proportional change on SR along the increasing LAI. Overall, RISR, SR and SASR are found to be the SVIs most sensitive to variation in LAI because they have shown high and large dynamic range of sensitivity (figure 4). This study suggests the use of simple adjustment of SR and ISR (i.e., resulting in SASR and RAISR, respectively) rather than modification using the local maximum and minimum for RSR and RISR to avoid a rather arbitrary decision for formulating the three-band SVIs. The simple adjustment and the local modifications result in comparable performance. On the contrary, SWIR-adjusted difference index (SADI) may be the most preferable SVI for the vegetation growing on various soil backgrounds because a consistent trend with LAI both in magnitude of the sensitivity and in the relationships along the increasing LAI is observed (figures 2(*f*) and 4). This means the same value of SADI from different soil backgrounds results in similar LAI. This is a very desirable attribute because when SVI is fitted empirically for LAI (LAI as a dependent variable), the equation results in a function that gives the same value of LAI output for the same value of SVI regardless of the soil background which it is derived from.

To evaluate the performance index shown in equations (8)–(10), a fifth-order polynomial least-squares function was fitted between the dependent variable Y (SVIs) and the independent variable X (LAI). The polynomial function was chosen to avoid the performance assessment dependency on the fitness of the function among the various SVIs with LAI shown in figure 2 because some exhibit purely logarithmic and the other sigmoidal relationships. Figure 5 shows the performance of the eight SVIs based on the Ji and Peters (2007) function (figure 5(*a*), equation (8)), the normalized Ji and Peters (2007) function (figure 5(*b*), equation (9)) and the new sensitivity function (figure 5(*c*), equation (10)). As stated before, equation (8) is a novel method yet does not show any confounding effects of the soil background; that is, for the same independent variable (LAI) the sensitivity and the standard error of the estimate of SVIs are the same regardless of the soil background. This indicates that this equation cannot be used on real canopies where soil background and other confounding factors can vary considerably or are unknown (Ji and Peters 2007). On the contrary, the new performance evaluation equation (10) works well on various confounding factors and on both linear and non-linear functions used to fit the relationships between Y and X.

From figure 5, SADI is the best performing index based on the consistency of the sensitivity along the various ranges of LAI and soil backgrounds. This is in good agreement with the consistency of the trend between SADI and LAI on various soil backgrounds (figures 2(*f*) and 4). There are compelling empirical reasons to use the normalized sensitivity of Ji and Peters (2009) function (equation (9)) instead of equation (8). SADI which was proven to be robust SVI in all of the other performance and sensitivity analyses including figure 5(*c*) was found to be the best index using equation (9) (figure 5(*b*)) instead of equation (8) (figure 5(*a*)). SR, RISR and SASR, the latter two being modifications of the SR, have shown relatively similar sensitivity to LAI (figure 4) which has resulted in a similar performance using equation (9) (figure 5(*b*)) instead of equation (8) (figure 5(*a*)).

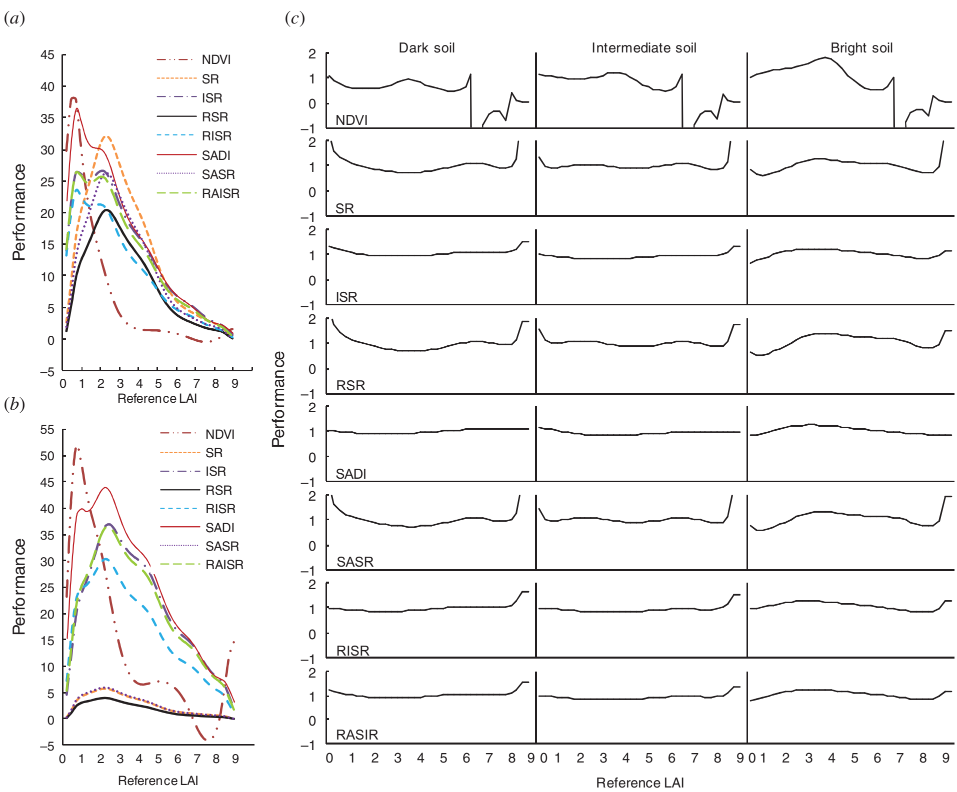

Figure 5. The performance of eight spectral vegetation indices following equation (8) (*a*), equation (9) (*b*), and equation (10) (*c*) plotted along LAI and various soil backgrounds. (*a*) and (*b*) when performance > 1.982 (essentially the same value with critical value in two-tailed *t*-test where $p = 0.05$ and degrees of freedom = 107), the SVIs are deemed to be sensitive to LAI; (*c*) when performance = 1, the SVIs are deemed to be highly sensitive to LAI.

NDVI was found to be the least performing index by all sensitivity and performance analyses (figure 5(*a*)). This is also in good agreement with Gonsamo (2010) who has shown NDVI to be an inappropriate index for semiempirical modelling of LAI.

3. Conclusions

Normalized sensitivity assessment in the cause-and-effect relationships between SVIs and spectral bands with LAI, and a new performance index are presented in this study, which are suitable for both linear and non-linear relationships. The latter was proven to be the robust performance index for SVI-based LAI estimation performance evaluations on the datasets simulated over varying soil backgrounds. However, any method is only as good as its ability to reproduce values from the standards that are considered the most ideal. Because of the lack of real datasets which cover a large range of LAI and soil backgrounds in this study, the performances of the new SVIs remain further to be evaluated. However, this study has shown that indices such as RSR and RISR can be adjusted without reflectances of local maximum and minimum in SWIR and R spectral bands, respectively. Several X and Y can be used to model the multi-parameter sensitivity analysis, such as various spectral bands with different land surface properties, as long as there is no interdependency between the independent (X_s) variables. SADI was found to be the best SVI for vegetation simulated on various soil backgrounds. It has also shown consistent trends of relationships with LAI on different soil backgrounds. Such a simple SVI as SADI could be a very reasonable indicator for studies targeting the amount of photosynthetic materials on land surface. The three-band (NIR–R–SWIR) SVIs have added merit compared to the commonly used two-band (NIR–R) SVIs when the background is heterogeneous or unknown if they are formulated in a way to better explain consistently the variable of interest (LAI) while suppressing the effect of other confounding factors. In general, the contribution of including the SWIR reflectance on the NIR–R-based SVIs is negligible for the vegetation simulated on the soil, which is characterized by high moisture content.

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